A Novel Adaptive Approach to Modeling MEMS Tunable Capacitors Using MRTD and FDTD Techniques

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Abstract — **This paper introduces a novel full wave technique for modeling MEMS tunable capacitors that is based on the coupling of physical motion of the MEMS device with Maxwell's equations through the modification of the MRTD/FDTD techniques. The difficulties of modeling MEMS devices are discussed, and ways to compensate for several of these are presented. The proposed approach is validated through comparison of simulation results to measurement for an interdigitated capacitor.**

I. Introduction

MEMS technology is quickly maturing and is considered to be very promising for use in RF devices. The lower loss characteristics, and thus higher Q factor, of MEMS devices make them very desirable for use as on-chip passives [1],[2]. MEMS passives have better performance because of their moving mechanical parts, although this makes them difficult to model. Existing full wave simulators model static devices, not moving ones, and have difficulties with small features and large aspect ratios. This paper presents a simulator, based on the MultiResolution Time-Domain (MRTD) [3] and Finite Difference Time-Domain (FDTD) [4] methods, that models electrostatically actuated interdigitated MEMS variable capacitors.

The FDTD and MRTD modeling methods are both full wave time domain techniques. In order to combine these methods with a motion simulation, a way must be found to couple the related quantities of the motion and electromagnetic equations, namely the position of the plates and the fields between them. The plates represent changing boundary conditions in the electromagnetic simulation. Both the plate position and electromagnetic field can be modeled in time, which allows the combination of a MEMS motion simulation with MRTD or FDTD.

II. MEMS Capacitor Modeling

The creation of a model of MEMS capacitors consists of two parts. The first is to find a motion model for the device and the second is to combine this model with an electromagnetic simulator (MRTD and FDTD). There are several similarities between the implementations using the two methods, however there are some critical differences that make MRTD the preferred choice for modeling MEMS devices.

A. Motion Modeling

A micrograph of the device being modeled is presented in Fig. 1. It can be considered a planar device for modeling purposes. The applied DC voltage causes an attractive force between the fingers. The fingers are on compliant flexures, and thus move closer together. As the spacing changes, the capacitance also changes. During operation, the RF field will propagate through the device, changing the voltage between the fingers and altering their separation. A model of the device will have to compensate for this effect.

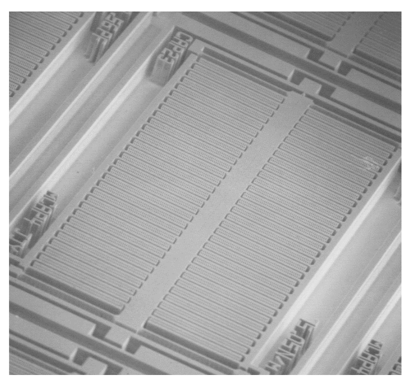

Fig. 1. MEMS Variable Capacitor

The modeling of the force between the fingers of the device in Fig. 1 can be a difficult electrostatic problem. However, the simplified model in Fig. 2 can be used as a first order approximation. Fig. 2 is a schematic for a parallel plate capacitor. The bottom plate is fixed and the spring and damper hold the top plate in place. The applied voltage causes an electrostatic attraction between the plates. This force causes the top plate to experience damped oscillatory motion. This motion can be represented mathematically.

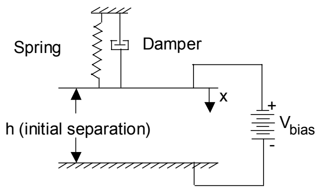

Fig. 2. Diagram of the simplified version of the MEMS parallel plate capacitor

The electrostatic force between the plates is:

$$f = \frac{\varepsilon_{\circ} A V^2}{(x-h)^2} \tag{1}$$

A is the area of the plates, V is the applied bias voltage, x is the displacement of the plate using the initial (V=0) position as zero, and h is the initial separation between the plates. The above can be used as the forcing function in the standard second order differential equation for spring mass systems [5]:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = \frac{\varepsilon_{\circ} A V^2}{(x-h)^2} \tag{2}$$

The coefficients in this equation represent the spring constant, k, the dampening coefficient, b, and the mass of the plate, m. Of course, the actual device does not have springs and dampers. The spring represents the elasticity of the flexure and the damper represents air resistance as well as nonlinearities in the elasticity of the flexure.

Equation (2) is an ordinary differential equation, and can be discretized using central differences. Using the standard notation:

$$u(n\Delta t) = u^n \tag{3}$$

(2) becomes:

$$x^{n+1} = 2x^n\left(\frac{2m - k\Delta t^2}{2m + b\Delta t}\right) + x^{n-1}\left(\frac{b\Delta t - 2m}{b\Delta t + 2m}\right) + \frac{2\Delta t^2 \varepsilon_{\circ} A V^2}{(2m + b\Delta t)(x^n - h)^2} \tag{4}$$

This equation can be used to determine the position of the plates due to an applied bias. It is assumed that the position is zero before the simulation begins. The applied voltage, V, can be constant (DC bias) or can vary with time (RF excitation). This facilitates the coupling with EM simulations.

B. Combination with Electromagnetic Models

In RF applications it is important to know the S-parameters of a device over a large frequency range. From these parameters other values can be determined, such as the capacitance or inductance of a device. In order to determine S-parameters over a large band, a full wave simulator is needed. FDTD and MRTD are excellent for this purpose. The time domain results provided by these methods can be converted to the frequency domain through a Fourier transform.

In a time domain simulation the metals represent boundary conditions. For perfect electrical conductors, electric field values tangential to the surface of the metal are set to zero. There is no requirement that these conditions are enforced at the same point for all time steps. In this manner, if the motion of a metal plate is known, it can be easily integrated into a simulation. The boundary conditions are simply enforced at each point on the metal at any given time step.

The position of the metal at any time step is given by (4). It is very similar in appearance to the expressions used in the FDTD technique. Using this expression, the position that the boundary conditions must be applied in the FDTD or MRTD simulation can be found. However, there are several issues that must be addressed with this implementation.

The first issue is that of representing the position of the plate in the FDTD or MRTD grid. The position of the plate is exact, not discretized. There are no points that can be set to be the same as in the electromagnetic grid. This creates a difficulty in the parallel plate case.

In FDTD there are two ways to address this issue. The first is to use a very fine grid in the area of the top capacitor plate. The position can be found exactly, and the boundary conditions can be applied at the nearest grid points to the metal. For the purpose of updating the position of the metal, however, the exact position value is used. This introduces inaccuracies that may be unacceptable.

The capacitance of a parallel plate capacitor can be expressed as:

$$C = \frac{\varepsilon_{\circ} A}{d} \tag{5}$$

As before, A is the area of the plates and d is the distance between them. From this expression, it is clear that capacitance is inversely proportional to the separation of the plates. In order for the effect of the capacitor to remain accurate a small discretization must be used. This introduces another restraint.

In order to keep an FDTD simulation accurate, the aspect ratio between the cell side lengths must not be very large. If the cell side length in the direction normal to the capacitor plate is small, the other cell side lengths must also be made small. In a parallel plate capacitor the aspect ratio between the plate width and separation can easily be 1000. The problem will quickly become computationally prohibitive.

Another solution of this problem is to move the grid points to the exact position of the plates during the simulation. In this manner, the spacing between the plates will be exact. However, this introduces error in representing the field at the new grid points. Interpolation must be used to determine the new field values, and the error introduced is not favorable.

In MRTD this is not as difficult. Because of the built in adaptive gridding of the method, it is ideal for simulating moving parts. Areas of high field variation are accurately represented by adding wavelets. Resolution in the area of the capacitor plates can be increased until the position of the plate is resolved to a desired threshold. This allows the accurate modeling of multiple conductors arbitrarily positioned in a cell [6]. It also has the added benefit of modeling the complex field structure at the edge of the capacitor very well. In this manner the capacitor can be modeled accurately.

Fig. 3 is a diagram illustrating the variable gridding of MRTD. It is a diagram of an interdigitated capacitor. The grid represented is for the time shown in Fig. 4, a field plot at a specific time step in an FDTD simulation. The white sections represent the highest intensity fields, the black are the lowest. The simulation method and more results are presented in the next section. The magnified sections in Fig. 3 show the gridding in each portion of the structure.

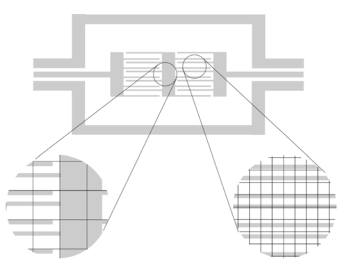

Fig. 3. Interdigitated capacitor used in simulations with magnified sections to demonstrate variable grid

In the center, the field variation is not very large. This is because, at this point in the simulation, most of the RF pulse is in the second set of fingers. Therefore, the discretization in this area is very coarse. In the second row of capacitor plates the field is varying quickly. The resolution in this area is much higher. The fine grid in this area is realized by adding wavelets to the field discretization.

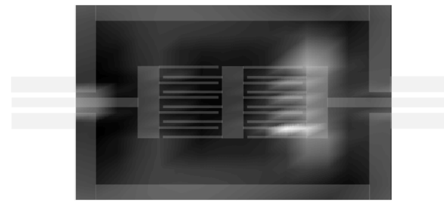

Fig. 4. Field plot with structure overlay, time step 1000 (1.527×10^{-10} seconds)

The interdigitated capacitor can also be modeled in FDTD. A modified discretization of Faraday's Law based on a contour path model provides a computationally efficient multi-finger model [4]. This method is based on approximating the amount of the cell boundary that is metallized, as well as the total metallized area of the cell. While this approach is not as accurate as MRTD, it is easier to implement.

The final problem in combining the simulations is the issue of the time step. The time step used in the mechanical simulation is significantly larger than that used in the electromagnetic simulation. Updating the position of the metal at each time step of the electromagnetic simulation is wasteful. Also, the movement of the plate in this time is negligible. Instead, the capacitor position will is every several hundred electromagnetic time steps.

III. Results

In order to test the simulation method suggested in this paper an interdigitated capacitor was modeled. The results presented here were generated using an FDTD simulator. In order to represent the complex geometry of the capacitor, the simulator utilized a static variable grid and the partial metallization approximation discussed above.

The schematic of the capacitor modeled is presented in Fig. 3. The capacitor consists of 10 sets of fingers arranged in two rows. When a bias is applied the fingers move closer together. The fingers and feeding structure have the same dimensions as the capacitor shown in Fig. 1. Fewer fingers were used in the simulation in order to reduce computation time. The results for the lower number of fingers were transformed in order to represent the complete capacitor.

In order to represent the structure, a variable grid was used in the direction transverse to signal propagation. In the area of the fingers, the grid alternated between 80 μm and 55.2 μm widths. Outside of the finger area, the grid spacing was 135.2 μm. The grid spacing in the other two directions was fixed at 186 μm. The variable grid approximation was applied in the area of the fingers. A PML was used on all sides. In the substrate below the capacitor, the PML represents an infinitely thick dielectric slab. The PML at the exit of the capacitor represents an infinitely long transmission line.

A Gaussian derivative pulse was introduced in the feed. The voltage between the signal and ground of the feed line was recorded. A field plot at time step 1000 is presented in Fig. 4. Once the input voltage was converted to the frequency domain, S_{11} was determined by the use of a simulation of a thru line with the same dimensions, on the same grid. The infinitely long transmission line on port 2 of the device acts as a resistor in series with the capacitor terminating the input line. The value of the resistor, 70.8Ω, is the characteristic impedance of the line.

The load impedance was determined and capacitance value extracted from S_{11}. A similar structure, consisting of only 3 sets of fingers per row was also simulated. By comparing the results of the two simulations, the capacitance of the full capacitor was determined.

A preliminary MRTD simulation was also performed using three wavelet resolutions. In the MRTD simulation, the number of cells was reduced by a factor of four by three by six. The absolute threshold used was 10^{-4}, the relative threshold used was 10^{-3}. The results were similar to those from the FDTD simulation.

The results for capacitance from the FDTD simulation are presented in Fig. 5. The simulated values are close to the measured value for the capacitor. A comparison is presented in Table 1. The measured values range from 1.4 to 1.6 pF in the 300 MHz to 1 GHz range, increasing with frequency. Experimental values for a given frequency have a tolerance of approximately 0.23pF. The variation comes from manufacturing tolerances as well as parasitic capacitance from the feeding pads. It is observed that capacitance increases with frequency. The increasing trend matches previously reported results [5].

IV. Conclusion

A method for modeling MEMS capacitors has been presented. Results from the simulation are similar to measured values from actual devices. The model presented herein uses the MRTD/FDTD techniques to produce results with a very high order of accuracy. This is very important for effective modeling of devices with complex geometries, such as MEMS capacitors.

Future directions for this work include accounting for metal loss in the simulation and adopting motion models with more degrees of freedom. When loss is accounted for, the Q of the device can be found. In addition, this technique could be expanded to model other types of MEMS devices.

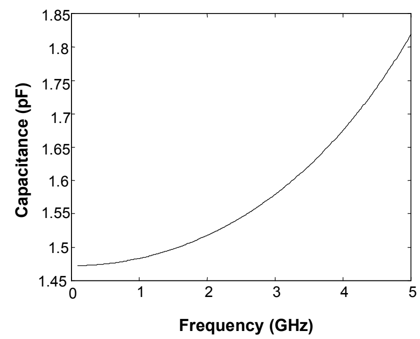

Fig. 5. Simulation Results: Capacitance vs. Frequency

Frequency (MHz)	Simulation (pF)	Measurement (pF)
208.105	1.46	1.58
403.475	1.46	1.62
500.00	1.47	1.4

Table 1. Comparison of simulated and measured capacitance values

Acknowledgement

The authors wish to acknowledge the support of the NSF CAREER Award, the Yamacraw Initiative of the State of Georgia, and the GT-Packaging Research Center.

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